# BOUNDARY LAYER FLOW OF NANOFLUID PAST AN INCLINED STRETCHING SHEET

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**ABSTRACT-***The problem of laminar fluid flow which results from an inclined stretching flat surface in nanofluid has been investigated numerically in this paper. The model used here for the nanofluid considers the effects of Brownian motion and thermophoresis. A similarity solution is presented which depends on the Prandtl number Pr, Lewis number Le, Brownian number Nb and thermophoresis number Nt. The coupled partial differential equations governing the flow are transformed into nonlinear boundary layer equations, which are then solved using the Nachtsheim-Swigert Shooting iteration technique along with the fourth order RungaKutta method. The variation of the Nusselt and Sherwood numbers with Nb ,Nt ,Pr , Le,Gr and Gc for various values of angle of inclinations α is presented in graphical forms.*

**Keywords:** *Boundary-layer, nanofluid, inclined stretching sheet, similarity solution.*

## INTRODUCTION

Nanotechnology has been widely used in industry since materials with sizes of nanometers possess unique physical and chemical properties. Nano-scale particle added fluids are called as nanofluid, which is firstly utilized by Choi [1]. Nano fluids represent an innovative way to increase thermal conductivity and, therefore, heat transfer. Unlike heat transfer in conventional fluids, the exceptionally high thermal conductivity of nanofluids provides for enhanced heat transfer rates, a unique feature of nanofluids. Advances in device miniaturization have necessitated heat transfer systems that are small in size, light mass, and high-performance[2]. Khan and Pop [3] have studied the problem of laminar fluid flow which results from the stretching of a flat surface in a nanofluid and investigated it numerically. All these and various other studies regarding nanofluids makes it clear the importance of nanofluid flows.

## MATHEMATICAL FORMULATION

The present problem considers the steady, nonlinear two-dimensional, mixed convective boundary layer flow of a viscous, incompressible nanofluid past an inclined stretching surface with an angle of inclination α with the horizontal and stretching with the linear velocity $u_w(x) = ax$, where a is a constant and x is the coordinate measured along the stretching surface. The sheet is stretched keeping the origin fixed when α is 0° and the variations are made accordingly with the change in inclination angles. The Cartesian coordinates (x, y) are chosen such that x-axis is chosen along the stretching sheet and y-axis is chosen perpendicular to it. It is assumed that at the stretching surface, the temperature T and the nanoparticle fraction C take constant values $T_w$ and $C_w$, respectively. The ambient values, attained as y tends to infinity, of T and C are denoted by $T_\infty$ and $C_\infty$, respectively. Here in this paper Copper-water nanofluid is the nanofluid chosen with water as the base fluid with Pr = 7.02 considering base temperature as 293 K. Solid spherical copper nano particles of 100 nm diameter mixed with water, has been considered as the nanofluid.

***Governing equations and boundary conditions***

The basic steady conservation of mass, momentum, thermal energyand nanoparticles equations for nanofluids can be written inCartesian coordinates x and y as, see Kuznetsov and Nield[4,5].The governing Prandtl boundary layer equations for the steady two dimensional laminar nanofluid flow over the inclined stretching surface are as follows:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=g\beta_{nf}(T-T_\infty)\sin\alpha+g\beta^*_{nf}(C-C_\infty)\sin\alpha -\frac{1}{\rho_{nf}}\frac{\partial p}{\partial x}+\nu_{nf}\left(\frac{\partial^2 u}{\partial y^2}\right) \tag{2}$$

$$0=g\beta_{nf}(T-T_\infty)\cos\alpha+g\beta^*_{nf}(C-C_\infty)\cos\alpha -\frac{1}{\rho_{nf}}\frac{\partial p}{\partial y} \tag{3}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_{nf}\left(\frac{\partial^2 T}{\partial y^2}\right)+\varepsilon\left[D_B\left(\frac{\partial C}{\partial y}\frac{\partial T}{\partial y}\right)+\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right] \tag{4}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B\left(\frac{\partial^2 C}{\partial y^2}\right)+\frac{D_T}{T_\infty}\left(\frac{\partial^2 T}{\partial y^2}\right) \tag{5}$$

Moreover, the boundary conditions for the velocity and temperature of this problem are:

$$\begin{aligned} &u=u_w(x)=ax\ ,v=0,T=T_{W,}\ C=C_W \quad at\ y=0\\ &u=0,T=T_\infty\ ,C=C_\infty\ as\ y\to\infty \end{aligned} \tag{6}$$

Here, u and v are the velocity components in x and y directions respectively, p is the perturbed fluid pressure, T is taken as the temperature, $\nu_{nf}$ is the kinematic viscosity of the nanofluid, $k_{nf}$ is the thermal conductivity, $(c_p)_{nf}$ is the specific heat, $\alpha_{nf}$ is the thermal diffusivity, $\beta_{nf}$ is the coefficient of thermal expansion of the nanofluid, $\beta^*_{nf}$ is the coefficient of volumetric coefficient of expansion with concentration, C is the nanoparticle volume fraction and $\rho_{nf}$ is the density of the nanofluid respectively. Here, a is a positive constant, $D_B$ is the Brownian diffusion coefficient, $D_T$ is the thermophoretic diffusion coefficient and $\varepsilon=\frac{(\rho c_p)_p}{(\rho c_p)_f}$ is the ratio between the effective heat capacity of the nanoparticle material and heat capacity of the fluid with $\rho$ being the density. Thus the governing Prandtl boundary layer equations for the steady two dimensional laminar nanofluid flow over an inclined stretching surface when 0° ≤ α ≤ 60° take the following form:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{7}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu_{nf}\frac{\partial^2 u}{\partial y^2}+g\,\beta_{nf}(T-T_\infty)\sin\alpha+g\,\beta^*_{nf}(C-C_\infty)\sin\alpha \tag{8}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_{nf}\left(\frac{\partial^2 T}{\partial y^2}\right)+\varepsilon\left[D_B\left(\frac{\partial C}{\partial y}\frac{\partial T}{\partial y}\right)+\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right] \tag{9}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B\left(\frac{\partial^2 C}{\partial y^2}\right)+\frac{D_T}{T_\infty}\left(\frac{\partial^2 T}{\partial y^2}\right) \tag{10}$$

**METHOD OF SOLUTION**

The following dimensionless variables are introduced in order to seek the solution of the problem:

$$\psi(x,y)=(a\nu_{nf})^{1/2}\,x\,f(\eta)\;,\;\eta=y\left(\frac{a}{\nu_{nf}}\right)^{1/2}, \tag{11}$$

$$\theta=\frac{T-T_\infty}{T_W-T_\infty}\;,\qquad \phi=\frac{C-C_\infty}{C_W-C_\infty}$$

where $\psi(x,y)$ is the stream function and velocity components can be expressed as

$$u=u_W f'(\eta) \tag{12}$$

$$v=-\left(a\nu_{nf}\right)^{1/2}f(\eta) \tag{13}$$

Accordingly, momentum, energy and concentration equations together with the boundary conditions, can be written as

$$f'''+ff''-(f')^2+\lambda\theta\sin\alpha+\lambda^*\phi\sin\alpha=0 \tag{14}$$

$$\frac{1}{(Pr)_{nf}}\theta''+f\theta'+N_b\phi'\theta'+N_t(\theta')^2=0 \tag{15}$$

$$\phi''+(Sc)_{nf}f\phi'+\frac{N_t}{N_b}\theta''=0 \tag{16}$$

with the corresponding boundary conditions

$$f(0)=0, f'(0)=1, \theta(0)=1, \quad \phi(0)=1$$
$$f'(\infty)=0, \theta(\infty)=0, \phi(\infty)=0 \tag{17}$$

Here primes denote differentiation with respect to η and $\nu_{nf}$ and $\alpha_{nf}$ are the kinematic viscosity and thermal diffusivity of the nanofluid respectively. Also, $(Pr)_{nf}$, $(Sc)_{nf}$, $N_b$ and $N_t$denote Prandtl number, Schmidt number, Brownian motion parameter and thermophoresis parameter respectively. Here, $(Re_x)_{nf}$ is the local Reynolds number, λ is the mixed convection parameter, λ* is the modified mixed convection parameter, $(Gr_x)_{nf}$ is the local Grashof number and $(Gc_x)_{nf}$ is the modified local Grashof number of the nanofluid.

## NUMERICAL ANALYSIS

The set of nonlinear differential equationssubject to the boundary conditions constitute the nonlinear boundary value problem. The above boundary value problem is converted into an initial value problem by shooting method. As the ordinary methods fail in order to solve the system of transformed equations together with the asymptotic boundary conditions, a special iteration technique is used. Equations are solved numerically subject to using Fourth-Order Runge-Kutta based shooting method along with Nachtsheim-Swigert iteration scheme for satisfaction of asymptotic boundary conditions. Initial guesses for the values of $f''(0)$ and $\theta'(0)$ are made to initiate the shooting process and these initial guesses are made taking into account of convergency and numerical results are obtained for several values of the physical parameter $(Pr)_{nf}$.

Concerning this study, quantities which are of interest are the non-dimensional skin friction coefficient $C_f$,Nusselt number $Nu$ and Sherwood number $Sh$ which are defined as

$$C_f = \frac{\tau_w}{\rho_f U_\infty^{\ 2}}, \text{ where } \tau_w = \mu_{nf}\left(\frac{\partial u}{\partial y}\right)_{y=0}$$

$$Nu = \frac{x q_w}{k_f (T_w - T_\infty)}, \text{ where } q_w = -k_{nf}\left(\frac{\partial T}{\partial y}\right)_{y=0}$$

$$Sh = \frac{x q_m}{D_B (C_w - C_\infty)}, \quad \text{where } q_m = -D_B\left(\frac{\partial C}{\partial y}\right)_{y=0}$$

where$q_w$ and $q_m$ are the wall heat flux and mass flux respectively. Kuznetsov and Nield(2009; 2010)referred $Nu\,(Re_x)_{nf}^{-1/2}$ and $Sh\,(Re_x)_{nf}^{-1/2}$ as reduced Nusselt number Nur $= -\frac{k_{nf}}{k_f}\theta'(0)$ and reduced Sherwood number Shr = $-\frac{(D_B)_{nf}}{(D_B)_f}\phi'(0)$ respectively.

## RESULTS AND DISCUSSION

The values of different parameters considered here are as follows: $\phi$ = 0.00,0.01,0.05,0.10 , Nt = 0.1,0.2,0.3,0.4,0.5 ,α = 0°,30°,45°and 60° and Pr = 6.06 (copper water nanofluid ) The following values are fixed : Le = 10,Nb = 0.1,Gr = 10,Gc = 10.

Fig.1 displays the effect of inclination angle α = 0°,30°,45°,60°on temperature distribution for specified parameters.When α increases, temperature decreases. From Fig.2. it is seen that as α increases, concentration decreases.InFig. 3,as α increases, dimensionless heat transfer rate also increases. It is shown in Fig.4 that as α increases, dimensionless concentration rate also increases for the same fixed values of Pr, Le,Nb,Nt,Gr and Gc.

## CONCLUSION

For copper-water nanofluid, as the inclination angle α increases, dimensionless temperature and dimensionless concentration decrease with increasing α.As the inclination angle α increases, reduced Nusselt number Nur and reduced Sherwood number Shrincreases respectively.

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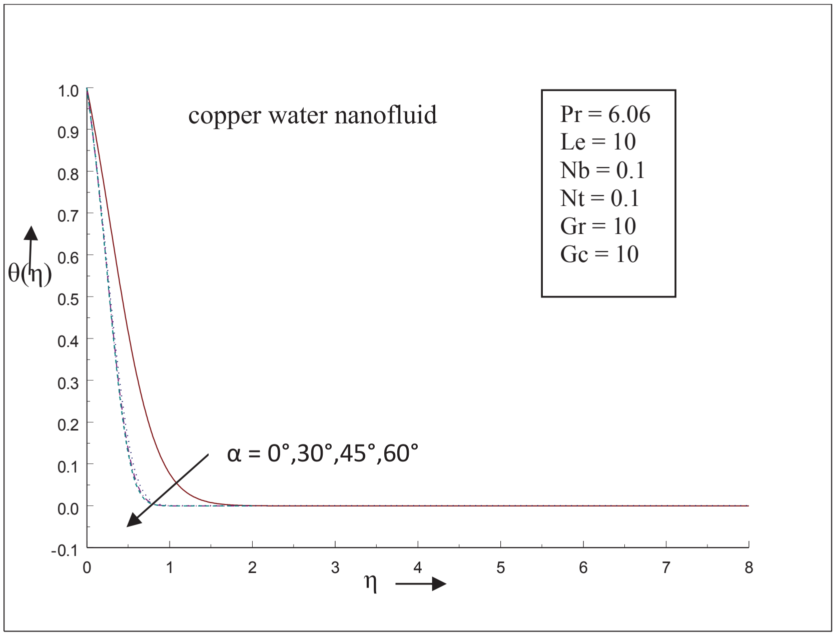


***Fig. 1***. Effect of inclination angle on temperature distribution for specified parameters for copper water nanofluid

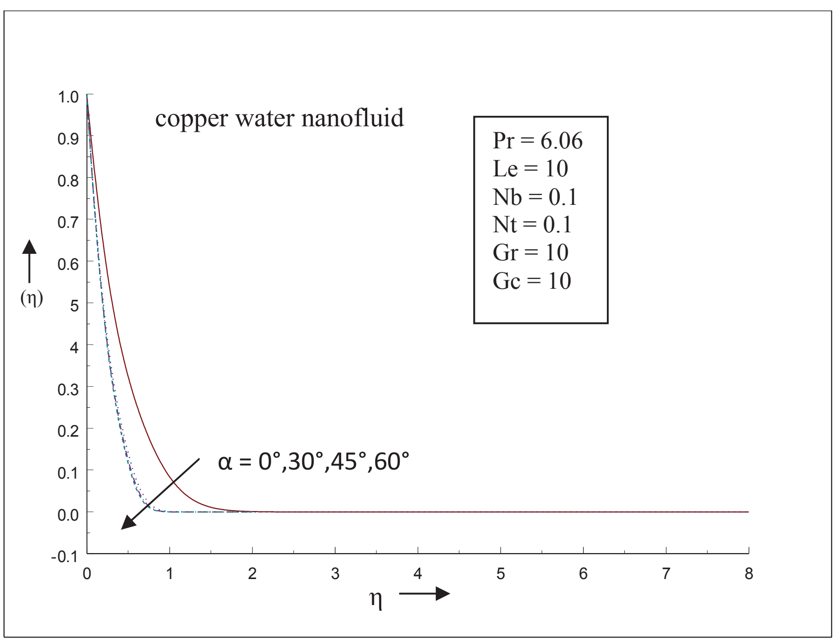


***Fig .2*** Effect of inclination angle α on concentration distribution for specific parameters for copper water nanofluid

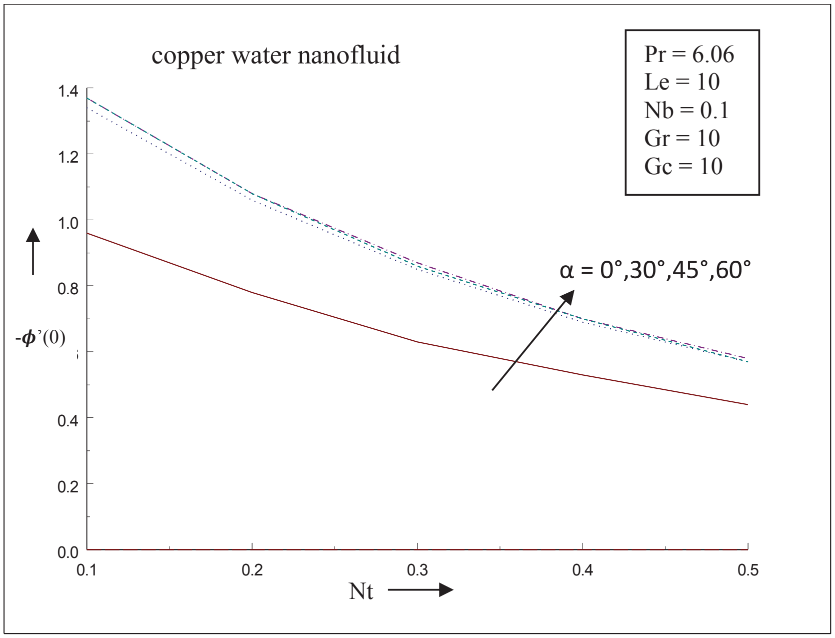


***Fig. 3***.Effects of inclination angles on dimensionless heat transfer rates for copper water nanofluid

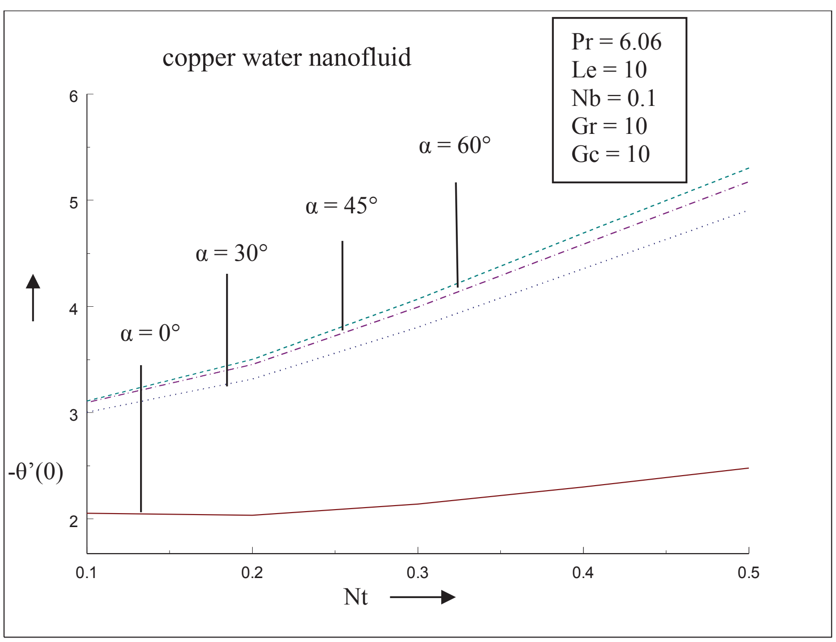


***Fig. 4*** Effects of inclination angles on dimensionless mass transfer rates for copper water nanofluid