

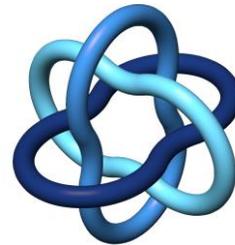
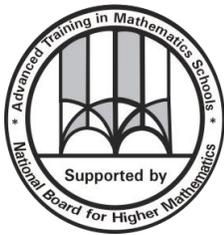


National Centre for Mathematics
A joint centre of IIT-B and TIFR-Mumbai



Workshop on Local Cohomology

St. Joseph's College
Irinjalakuda, Kerala, India



June 20 - July 2, 2016

Speakers

H. Ananthnarayan (IIT Bombay)
Markus Brodmann (University at Zurich)
N. T. Cuong (Institute of Mathematics, Hanoi)
Krishna Hanumanthu (CMI, Chennai)
Le Tuan Hoa (Institute of Mathematics, Hanoi)
Manoj Kummini (CMI, Chennai)
Dilip Patil (IISc, Bangalore)
Tony J. Puthenpurakal (IIT Bombay)
Peter Schenzel (Martin-Luther-University at Halle-Wittenberg)

Organisers

Markus Brodmann, Sr. Christy, Mangala Thottuvai, Jugal K. Verma

Academic Report

A workshop/conference on Local Cohomology was organized in St. Joseph's College, Irinjalakuda, Kerala from 20 June 2016 to 02 July 2016 with the support of National Centre for Mathematics (A joint venture of TIFR and IIT Mumbai), National Board for Mathematics (DAE) and International Mathematical Union, Germany.



Inaugural Session

The thirteen day workshop/conference was inaugurated in the New Seminar Hall, St. Joseph's College, Irinjalakuda, Higher Educational Institution in the birth place of the fourteenth century great Mathematician-Astronomer Samgama Grama Madhava by the eminent Mathematician Prof. Peter Schenzel, University of Halle. Prof. Jugal Verma(Dean IIT Mumbai), Dr.Sr. Lilly P L (Principal, St. Joseph's College, Irinjalakuda), Dr. N R Mangalambal(Head, Department of Mathematics, St. Joseph's College, Irinjalakuda) were the organizers. The inaugural ceremony was conducted in the traditional way with a recital on Edakka.

Report on the School

Program Highlights

The theory of local cohomology modules was introduced by A. Grothendieck. The Publication of R. Hartshorne's lecture notes of Grothendieck's Harvard seminars established the importance of local cohomology as an effective tool to solve many problems in commutative algebra and algebraic geometry. This workshop introduced basics and advanced aspects of this theory along with major applications and recent developments. The speakers established necessary background in commutative algebra to train the participants in the first week so that they could follow advanced lectures in the second week. Some of the participants also presented their works on local cohomology.

There were 37 lectures, 17 tutorials and 11 paper presentations

Speakers

- H. Ananthnarayan (IIT Bombay)
- Markus Brodmann (University at Zurich)
- N. T. Cuong (Institute of Mathematics, Hanoi)
- Krishna Hanumanthu (CMI, Chennai)
- Le Tuan Hoa (Institute of Mathematics, Hanoi)
- Manoj Kummini (CMI, Chennai)
- Dilip Patil (IISc, Bangalore)
- Tony J. Puthenpurakal (IIT Bombay)
- Peter Schenzel (Martin-Luther-University at Halle-Wittenberg)
- J K Verma(IIT Bombay)

Report of Course in Brief

1. Manoj Kummini (CMI, Chennai)

Manoj Kummini delivered 3 lectures on the following topics:
Injective modules: Injective Modules, existence of injective hull, structure theorem for irreducible injective modules, examples of injective modules, Matlis duality.



Manoj Kummini guided 6 tutorials.



2. Tony Puthenpurakal (IIT Bombay)

Tony Puthenpurakal delivered 7 lectures on the following topics:

Local cohomology: Resolutions and derived functors, Local Cohomology functor, direct limit of Ext modules, direct limit of Koszul cohomology modules. Depth, non-vanishing and vanishing theorems. Ring of differential operators on $K[[X_1 \dots X_d]]$ where K is a field of characteristic zero. Application to local cohomology modules over Regular rings containing a field of characteristic zero. Associated primes of local cohomology modules. Finiteness of Bass numbers of local cohomology modules. Bounds on injective dimension of local cohomology modules.



Tony Puthenpurakal conducted 11 tutorials.



3. Dilip Patil (IISc, Bangalore)

Dilip Patil delivered 4 lectures on the following topics:

Cohen-Macaulay rings: Koszul complex, regular sequences, characterisation of depth via Ext modules, Cohen-Macaulay modules and their characterisations, unmixedness theorems, examples of CM rings, Auslander-Buchsbaum formula. Ferrand-Vasconceos theorem about $I=I^2$.



Dilip Patil conducted 8 tutorials.



4. H. Ananthnarayan (IIT Bombay)

H Ananthnarayan delivered 4 lectures on the following topics:
Gorenstein rings: various definitions of Gorenstein rings, Bass numbers, injective resolutions, canonical module, local duality, examples of Gorenstein rings, Serre's theorem.



H Ananthnarayan conducted 5 tutorials.



5. Krishna Hanumanthu (CMI, Chennai)

Krishna Hanumanthu delivered 3 lectures on the following topics:

Serr'e FAC paper: : Sheaves, Cech cohomology, Flasque sheaves, relation between local cohomology and sheaf cohomology, Grothendieck-Serre difference formula for Hilbert functions, projective Varieties: graded local cohomology, sheaves on projective varieties, global sections and cohomology.



Krishna Hanumanthu conducted 2 tutorials.



6. Markus Brodmann (Univ. Zurich)

Markus Brodmann delivered 4 lectures on the following topics:

D-modules: Filtered algebras, derivations, Weyl algebra, the standard basis, weighted degrees and filtrations, weighted associated graded rings, filtered modules, D-modules, Grobner bases, weighted orderings, standard degree and Hilbert polynomials.



Markus Brodmann guided 5 tutorials



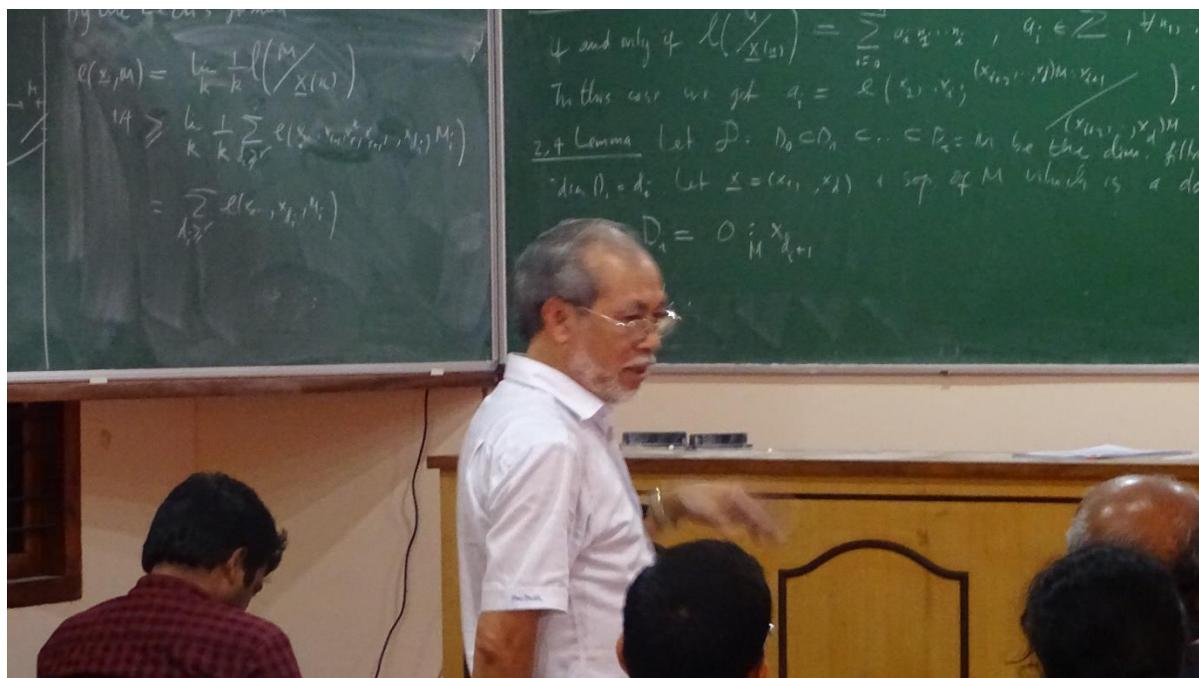
7. N.T. Cuong (Institute of Mathematics, Hanoi)

N T Cuong delivered 4 lectures on the following topics:

Sequentially Cohen-Macaulay modules: Filtrations and good systems of parameters, sequentially Cohen-Macaulay modules, characterization of sequentially Cohen-Macaulay modules by good systems of parameters.



N T Cuong conducted 4 tutorials.



8. Le Tuan Hoa (Institute of Mathematics, Hanoi)

Le Tuan Hoa delivered 4 lectures on the following topics:

Castelnuovo-Mumford regularity: Characterisations of Castelnuovo-Mumford regularity via local cohomology, Mumford's characterization of regularity of sheaves on projective space and its extension by Eisenbud-Goto, regularity and reduction numbers of large powers of ideals, bounding regularity in terms of degrees of defining equations, regularity and Hilbert coefficients, complexity of Grobner bases, some open problems.



Le Tuan Hoa conducted 4 tutorials



9. Peter Schenzel (Univ. Halle)

Peter Schenzel delivered 3 lectures on the following topics:

Bezout's Theorem and local cohomology: Rees and form rings, Hilbert function, Hilbert polynomial, multiplicities, Auslander-Buchsbaum-Serre formula for multiplicities of parameter ideals, subcomplexes of Koszul and Cech-complexes, inequalities for local intersection numbers in Bezout's, theorem, corrections of results of Pritchard and Greuel-Lossen-Justin.



Peter Schenzel conducted 6 tutorials



10. J. K. Verma (IIT Bombay)

J. K. Verma delivered 1 lecture on the following topic:
Stanley's solution of the ADG conjecture.



Report on Research Talks

1. Extremal Rays of the Betti Cones

Rajiv Kumar
Indian Institute of Technology Bombay

In 2008, Boij and Soderberg stated a conjecture for the extremal rays of the Betti cone of modules over a polynomial ring over a field. This was proved in 2009 by Eisenbud and Scheryer. A similar characterisation was proved in 2012 for a quadratic hypersurface of embedding dimension 2 by Berkesch, Burke, Erman and Gibbons. Generalising the second result, we find a class of one dimensional rings where the extremal rays are exactly the ones spanned by the Betti tables of shifts of $R/R(1)$ and R/m^n , where l is a regular element and $n \geq 1$. We also give an example of a one-dimensional ring to show that the above result does not hold in general.



2. Behaviour of Finiteness of the Set of Associated Primes of Lyubeznik Functor under Ring Extensions

Rajsekhar Bhattacharyya
Dinabandhu Andrews College, Garia, Kolkata

We study the behaviour of the finiteness of the set of associated primes of local cohomology modules, more generally of Lyubeznik functors, under various ring extensions. At first, we review the results for at and faithfully at extensions and we present new applications of them. Then, we show that the finiteness property can be transferred from a ring to its pure local subring. Further, we observed that under mild conditions on the rings, finiteness property can come down from cyclically pure ring extensions to its local base ring. In particular, we observe that the set of associated primes of Lyubeznik functors of a cyclically pure local subring (which turns out to be Cohen-Macaulay) of equicharacteristic or unramified regular local ring, is finite.



3. Asymptotic linear bounds of Castelnuovo-Mumford regularity in multi-graded modules

Dipankar Ghosh

Indian Institute of Technology Bombay

Let A be a Noetherian standard \mathbb{N} -graded algebra over an Artinian local ring A_0 . Let $I_1 \dots \dots I_t$ be homogeneous ideals of A and M a finitely generated \mathbb{N} -graded A -module. We prove that there exist two integers k_1 and k'_1 such that

$$\text{reg}(I_1^{n_1} \dots \dots I_t^{n_t} M) \leq (n_1 + \dots + n_t)k_1 + k'_1 \text{ for all } n_1, \dots, n_t \in \mathbb{N}$$



We prove this result in a quite general set-up. As a consequence, we also obtain the following: If A_0 is a field, then there exist two integers k_2 and k'_2 such that

$$\text{reg}(\bar{I}_1^{n_1} \dots \dots \bar{I}_t^{n_t} M) \leq (n_1 + \dots + n_t)k_2 + k'_2 \text{ for all } n_1, \dots, n_t \in \mathbb{N}$$

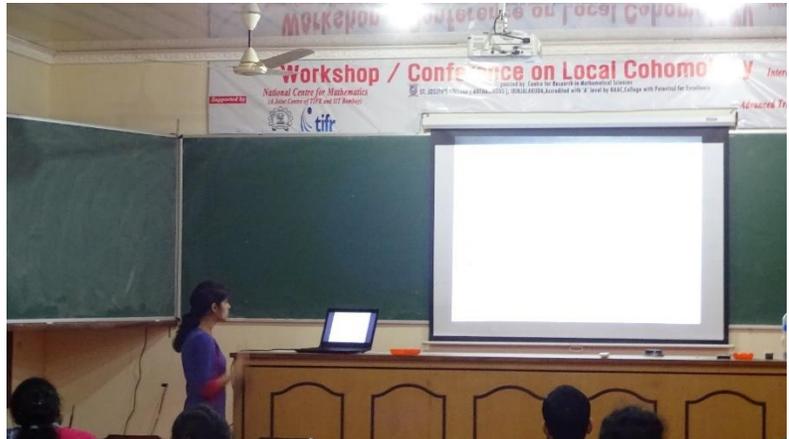
where \bar{I} denotes the integral closure of an ideal I of A :

4. Decomposing Gorenstein Rings as Connected Sums

Jai Laxmi

Indian Institute of Technology Bombay

In 2012, Ananthnarayan, Avramov and Moore give a new construction of Gorenstein rings from two Gorenstein local rings, called their connected sum. Given a Gorenstein Artin ring, one would like to know whether it decomposes as a connected sum and if so, what are its components. We will first give conditions for decomposability, and will further show that indecomposable components appearing in the decomposition are unique up to isomorphism.



5. Local cohomology of multi-Rees algebras, joint reduction numbers and product of complete ideals

Parangama Sarkar
Indian Institute of Technology Bombay

We find conditions on local cohomology modules of multi-Rees algebras of multi-graded admissible filtrations of ideals which enable us to predict joint reduction numbers. As a consequence, we are able to prove a generalization of a result of Reid-Roberts-Vitulli in the setting of analytically unramified local rings for completeness of power products of complete ideals.



6. Hilbert Kunz Multiplicity and Hilbert Kunz density function of Segre product and tensor product of rings

Mandira Mondol
Tata Institute of Fundamental Research

For a Noetherian local (R, \mathfrak{m}) or for a Noetherian graded k -algebra R and an ideal $I \subset R$ such that $l(R/I)$ is finite and characteristic of $R = p > 0$, the Hilbert-Kunz function is defined as $HK(R, I)(q) = l(R/I^{[q]})$, where $q = p^n$ and $I^{[q]}$ denotes the n th Frobenius power of I . In early 80's

Monsky has proved that, if d denotes the dimension of R then

$$HK(R, I)(q) = e_{HK}(R, I)q^d + O(q^{d-1}),$$

where $e_{HK}(R, I)$, called *HK multiplicity*, is a positive real number. Over the years, Hilbert-Kunz multiplicity for different rings has been studied by Monsky, Conca, Brenner, Trivedi, Watanabe, Yoshida and many others. To study e_{HK} in a recent paper Trivedi has introduced a new invariant for a pair (M, I) , where R is a standard graded Noetherian ring of dimension d over an algebraically closed field k of char $p > 0$, I is a homogeneous ideal of finite colength and M is a finitely generated graded R -module, called *the Hilbert Kunz density function of*

(M, I) [denoted $HKd(M; I)$]. She discussed HKd functions, and as a consequence, the e_{HK} of Segre product and Tensor product of finitely many rings.



7. On Hilbert coefficients of parameter ideals and Cohen-Macaulayness

Kumari Saloni

Indian Institute of Technology Guwahati

Let (R, m) be a Noetherian local ring, Q a parameter ideal and K an m -primary ideal containing Q . The Hilbert coefficients of Q with respect to K , namely $g_i^K(Q)$ which arise from the function $l(R/KQ^n)$ contain considerable information about the properties of the ring R . They discussed a necessary and sufficient condition for an unmixed local ring R to be Cohen-Macaulay in terms of $g_0(Q)$ and $g_1(Q)$. As a consequence, we obtain a result of Ghezzi et al. which settles the negativity conjecture of W. V. Vasconcelos in unmixed local rings.



8. de Rham Cohomology of Local Cohomology modules

Rakesh Reddy

Indian Institute of Science Education and Research, Trivandrum

Let K be a field of characteristic zero and $R = K[x_1, x_2, \dots, x_n]$ be the polynomial ring in n variables with coefficients in K . Let I be an ideal in R . Let $H_I^i(R)$ denotes the i^{th} local cohomology module. Let $A_n(K)$ be the n th Weyl algebra. For an $A_n(K)$ module N let $H^i(\partial; N)$ denote the de Rham cohomology module of N with respect to $\partial = \partial_1, \dots, \partial_n$.

In one of his remarkable papers Lyubeznik proved that for each i , $H_I^i(R)$ are holonomic $A_n(K)$ modules. In general de Rham cohomology modules of a holonomic $A_n(K)$ module are only finite dimensional K vector spaces. Recently Tony J. Puthenpurakal related $\dim_K H^*(\partial; H_I^*(R))$ with some properties of the ideal I . In this talk we study Euler-characteristic of curves, Cohen-Macaulay surfaces and non-singular surfaces. We relate Euler-characteristic of the curves with its degree. We also give a lower bound for $\dim_K H^n(\partial; H_P^{n-1}(R))$ in terms of number of points at infinity of $V(P)$, where P is a prime ideal of height $n-1$. For Cohen-Macaulay surfaces we relate the Euler-characteristic with the cohomological dimension of the ideal. Finally we derive the Euler-characteristic of non-singular surfaces in terms of Bass numbers of local cohomology modules of the ideal.



9. Coisotropic Property of Characteristic Varieties

Jyoti Singh

Indian Institute of Technology Bombay

An important geometric invariant in the theory of D-modules is the characteristic variety.

Purpose of this talk is to describe the coisotropic property of characteristic variety of a module over Weyl algebras with some examples. Let $A_n(\mathbb{C})$ be the Weyl algebra and M a finitely generated left $A_n(\mathbb{C})$ -module. Let $Ch(M)$ denote the variety corresponding to the zeros of the annihilator of $gr^F M$ of \mathbb{C}^{2n} . It is an invariant of M , called its characteristic variety. We show that characteristic variety of a finitely generated $A_n(\mathbb{C})$ -module with Bernstein is always coisotropic. In support of this, we give some examples of modules over Weyl algebra and their characteristic varieties which are coisotropic.



10. Castelnuovo-Mumford Regularity

Clare D'Cruz

Chennai Mathematical Institute

Does the regularity increase if we pass to the radical or remove embedded primes? By examples, we show that this happens. As a by-product we are also able to answer some related questions. In particular, we provide examples of licci ideals related to monomial curves in P^3 (resp. in P^4) such that the regularity of their radical is essentially the square (resp. the cube) of that of the ideal.



11. Regularity of Binomial Edge Ideals of Block Graphs

A. V. Jayanthan

Indian Institute of Technology Madras

Given a finite simple graph, connections between the combinatorial invariants of the graph and the algebraic invariants of the corresponding edge ideals have been an active area of research for the past two decades. In this joint work with Narayanan and Raghavendra Rao, we study the regularity of binomial edge ideals of certain classes of graphs. We obtain an improved lower bound for the regularity of the binomial edge ideals of trees. We prove an upper bound for the regularity of the binomial edge ideals of certain subclass of block-graphs. As a consequence we obtain sharp upper and lower bounds for the regularity of binomial edge ideals of a class of trees called lobsters. We also obtain precise expressions for the regularities of binomial edge ideals of certain classes of trees and block graphs.



Schedule of Lectures and Tutorials

Day	Date	9.30	11.00	11.30	1.00	2.00	3.30	4.00-4.50 (tut)	4.50	5.10-6.00 (tut)
Mon	20-6	DP	Tea	TP	Lunch	MK	Tea	DP, mk, tp	snacks	MK,dp,tp
Tue	21-6	DP		TP		MK		TP, mk, dp		MK, dp, tp
Wed	22-6	DP		TP		MK		DP, tp, mk		MK, dp, tp
Thu	23-6	DP		TP		PS		TP, dp, ps		PS, dp, tp
Fri	24-6	MB		HA		PS		MB, ha, ps		HA, mb, ps
Sat	25-6	MB		HA		PS		HA, mb, ps		PS, mb, ha
Sun	EXCURSION									
Mon	27-6	MB	Tea	HA	Lunch	KH	Tea	MB,ha,kh	snacks	CD, DG
Tue	28-6	MB		HA		KH		JKV		JS, RR
Wed	29-6	LH		NC		KH		LH, nc,kh		PS, KS
Thu	30-6	LH		NC		TP		NC, lh,tp		MM, RB
Fri	.1-7	LH		NC		TP		LH, nc, tp		JL, RK
Sat	.2-7	LH		NC		TP		NC, lh, tp		Val

Clare D'Cruz (CD)	CMI, Chennai	Castelnuovo-Mumford regularity
Dipankar Ghosh (DG)	IIT Bombay	Asymptotic linear bounds of Castelnuovo-Mumford regularity in multi-graded modules
J. K. Verma (JKV)	IIT Bombay	Stanley's solution of the ADG conjecture
Jyoti Singh (JS)	IIT Bombay	Coisotropic Property of Characteristic Varieties
Rakesh Reddy (RR)	IISER, Trivandrum	de Rham Cohomology of Local Cohomology modules
Parangama Sarkar (PS)	IIT Bombay	Local cohomology of multi-Rees algebras & joint reduction numbers and product of complete ideals
Kumari Saloni (KS)	IIT Guwahati	On Hilbert coefficients of parameter ideals
Mandira Mondol (MM)	TIFR, Mumbai	Hilbert Kunz Multiplicity and Hilbert Kunz density function of Segre product and tensor product of rings
R. Bhattacharyya (RB)	DA College, Kolkata	Behaviour of μ -finiteness of associated primes of Lyubeznik functor under ring extensions
Jai Laxmi (JL)	IIT Bombay	Decomposing Gorenstein Rings as Connected Sums
Rajiv Kumar (RK)	IIT Bombay	Extremal Rays of the Betti Cones
A.V. Jayanthan (AVJ)	IIT Madras	Regularity of binomial edge ideals of block graphs

Speaker	Topic	Scribe
Dilip Patil	Cohen-Macaulay rings	Rajiv Kumar
Tony Puthenpurakal	Local Cohomology	Sudheshna Roy
Tony Puthenpurakal	D- modules and Local Cohomology	Rakesh Reddy
Manoj Kummini	Matlis Duality	Kriti Goel
Markus Brodmann	D-Modules	H. Ananthnarayan
H. Ananthnarayan	Gorenstein rings	Jai Laxmi
Peter Schenzel	Bezout's Theorem and Local cohomology	Clare D'cruz
L.T. Hoa	Castelnuovo-Mumford regularity	Dipankar Ghosh
N. T. Cuong	Sequentially Cohen-Macaulay modules	Parangama Sarkar
Krishna Hanumanthu	Serre's FAC	Mandira Mondal



List of Participants

Sr.	Full Name	Affiliation	Position in College/ University
1	Ms. Kriti Goel	IIT Bombay	PhD
2	Mr. Rajiv Kumar	IIT Bombay	PhD
3	Ms. Sudeshna Roy	IIT Bombay	PhD
4	Ms. Jai Laxmi	IIT Bombay	PhD
5	Mr. Dipankar Ghosh	IIT Bombay	PhD
6	Dr. Jyoti Singh	IIT Bombay	PDF
7	Ms. Parangama Sarkar	IIT Bombay	PhD
8	Mr. Rajib Sarkar	CMI	PhD
9	Ms Mitra Koley	CMI	PhD
10	Ms. Clare D'Cruz	CMI	Associate Professor
11	Mr. Rijul Saini	TIFR	PhD
12	Mr. Sujoy Chakraborty	TIFR	PhD
13	Mr. Sampat Kumar Sharma	TIFR	PhD
14	Mr. Amith K Shastri	TIFR	PhD
15	Mr Ravitheja Vangala	TIFR	PhD
16	Ms Mandira Mondal	TIFR	PhD
17	Mr. Subhajit Chanda	IIT MADRAS	PhD
18	Mr. Selvaraja S	IIT MADRAS	PhD
19	Mr. Ram Shila	JNU	PhD
20	Mr. Joydip Saha	RKMVU	PhD
21	Mr. Ajay Kumar	Dav University Jalandhar	Assistant Professor
22	Dr. Rajsekhar Bhattacharyya	Dinabandhu Andrews College	Associate Professor
23	Mr. Soumitra Das	North-Eastern Hill University	PhD
24	Mr. Prabakaran P	Periyar University	PhD
25	Mr.Bibekananda	University Of Kansas, USA	PhD
26	Dr. Rakesh Reddy	IISER, Trivandrum	Assistant Professor
27	Ms. Sabna K S	St. Joseph's College	Assistant Professor
28	Ms. Dhanya V S	St. Joseph's College	Assistant Professor
29	Ms. Kumari Saloni	IIT Guwahati	PhD
30	Mr. Anoop Singh	HRI	PhD

Valedictory



The valedictory function was presided over by Dr.Sr. Annie Kuriakose, Education Counselor of Pavanathma Educational Society to which St. Joseph's College belong and former Principal. The certificates were distributed during the function. The speakers and participants shared their experience. This programme was ended with a high-tea.