



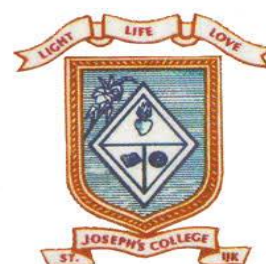
National Centre for Mathematics
A joint centre of IIT-B and TIFR-Mumbai



Instructional School for Teachers On Commutative Algebra



St. Joseph's College [Autonomous]
Irinjalakuda, Kerala, India



April 24 - May 6, 2017

Speakers

ManojKummini (CMI, Chennai)
Clare D'Cruz (CMI, Chennai)
Krishna Hanumanthu (CMI, Chennai)
Sarang Sane (IIT Madras)

Organisers

ManojKummini, Clare D'Cruz, Mangalambal N R

Academic Report

An **Instructional School for Teachers on Commutative Algebra** was organized in St. Joseph's College [Autonomous], Irinjalakuda, Kerala from 24 April 2017 to 06 May 2017 with the support of National Centre for Mathematics (A joint venture of TIFR and IIT Mumbai) and National Board for Higher Mathematics (DAE)



Inaugural Session

The thirteen day Instructional School for Teachers was inaugurated in the New Seminar Hall, St. Joseph's College [Autonomous], Irinjalakuda, in the birth place of the fourteenth century great Mathematician-Astronomer Samgama Grama Madhava by Dr. Manoj Kummini (CMI, Chennai). Dr. Sr. Lilly P L (Principal, St. Joseph's College, Irinjalakuda) presided over the meeting and Dr. N R Mangalambal (Head, Department of Mathematics, St. Joseph's College, Irinjalakuda) offered a formal welcome.

Report on the School

This was an introductory school in commutative algebra with topics and problems from the text books Atiyah-Macdonald, *Introduction to Commutative Algebra*, Eisenbud, *Commutative Algebra with a View towards Algebraic Geometry* and Gopalakrishnan, *Commutative Algebra*. There were 24 lectures and 24 tutorials.

Speakers

- ManojKummini (CMI, Chennai)
- Clare D'Cruz (CMI, Chennai)
- Krishna Hanumanthu (CMI, Chennai)
- Sarang Sane (IIT Madras)

Tutors

- ManojKummini (CMI, Chennai)
- Clare D'Cruz (CMI, Chennai)
- Krishna Hanumanthu (CMI, Chennai)
- Sarang Sane (IIT Madras)
- Suhas M B (IIT Madras)
- MitraKoley (CMI, Chennai)
- Praveen Kumar Roy (CMI, Chennai)

Report of Course in Brief



1. ManojKummini, CMI, Chennai

- Ring and morphisms-There was some confusion about why we define ring morphisms the way we do (with $1 \mapsto 1$). ManojKummini gave examples from rings of continuous functions and later interpreted polynomial rings as rings of functions (from \mathbb{R} or \mathbb{C} , so they can think of these as continuous functions).He gave many concrete examples of ring morphisms (over \mathbb{R} and \mathbb{C}) and what they mean geometrically,
- Hilbert Basis Theorem. It was stated without proof; the proof in AtiyahMacdonald is quite straight-forward.
- Nullstellensatz. It was used (without proof) to give many examples of geometric interest.
- Spectrum. Manoj defined prime spectrum Spec , and showed that contraction gives a continuous map in the Zariski topology and sketched that in artinian rings, all prime ideals are maximal and that there are only finitely many maximal ideals.

2. Sarang Sane, IIT Madras



- **Sarang Sane** talked about tensor products, localization and exact sequences after some preliminary discussion and examples of modules. In particular, he defined and covered basic notions of modules, submodules, operations on them, free modules, generating sets for modules, basis for a free module, sums, direct sums, intersections, etc. Some time was spent on furthering the example of continuous functions by talking about continuous functions from X to \mathbb{R}^n and proving that it is a (free) module over the ring of continuous functions from X to \mathbb{R} . He then looked at the case of X a sphere and considered those submodules of those functions which give orthogonal vectors at each point (tangent bundle) and parallel (normal bundle). Made them prove that they are submodules and that their direct sum is the free one. For the circle, they checked that both these submodules are themselves free and the parallel
- About tensor products, Sarang stated the construction and the universal property, and the participants did several exercises where they had to define module homomorphisms, and after initially struggling, it may have sunk in that one has to use the universal property or the construction to define these. He stated several times that $M \otimes R/I \cong M/IM$ and that tensor is right exact. About exact sequences, he defined exactness, short exact sequences and the notion of a free resolution. The participants haven't solved any exercises of note on this.
- About localization, he defined it, and the participants got a feel about how to view it (solved several problems). He also proved the property that localization captures when a module is 0 (and hence injectivity and surjectivity of maps). We also saw that localization preserves exact sequences and that only ideals disjoint from the multiplicative set are preserved. In particular that prime ideals in the localization are in bijective correspondence with the prime ideals not intersecting the multiplicative set. Using this, he also proved that rank is well-defined and that a generating set for any finite rank free module with the same size as the rank must be a basis. Finally, Sarang also proved prime avoidance.

3. Clare D'Cruz, CMI, Chennai



- Definition of primary ideals, Primary ideals and localization, First Uniqueness Theorem for Primary Decomposition with examples, Associated Primes, Second Uniqueness Theorem for Primary Decomposition, Primary Decomposition in Noetherian Rings, Examples of ideals in Noetherian rings where the minimal primary components were computed using Second Uniqueness theorem, Symbolic Powers, Examples of Artin rings,
- The tutorial sessions consisted of: Computing Primary components corresponding to minimal primes, Associated primes, Primary Decomposition in certain Examples, Symbolic powers, Example of a non-Noetherian ring, Examples of rings which satisfy acc and dcc, and Introduction to Macaulay - (Rings, Ideals, Edge Ideals, Primary Decomposition)

4. Krishna Hanumanthu, CMI, Chennai



- Integral extensions - definition, characterization in terms of finite generation as modules; examples like the cusp, node, $x^2 - y^2$ over the x -axis; hyperbola over the x -axis (i.e., $xy - 1$) and the union of two axes over x -axis (i.e., xy). First one is integral and the the last two are not. He indicated why integrality fails (empty fibre, infinite fibre) in these two examples.
- Going-up theorem and all its manifestations: lying over theorem, spec map is surjective, closed etc. Krishna also proved that the number of primes lying over a fixed prime is finite.
- Valuation rings: Proved all the results leading to showing that integral closure of A in a field K is the intersection of all valuation rings of K containing A . Using the results on valuation rings, proved the version of Nullstellensatz that Manoj mentioned: a field K which is finitely generated as an algebra over a field F is a finite extension of F . Also gave four other versions of Nullstellensatz and proved the first two of them: (1) maximal ideals in $k[x_1, \dots, x_n]$ are in bijection with k^n ; (2) every proper ideal in $k[x_1, \dots, x_n]$ has zeroes; (3) $I(V(I)) = \text{rad}(I)$; all three versions above over an alg closed field k ; and finally (4) radical of an ideal in a finitely generated algebra over a field is the intersection of maximal ideals. Noether normalization: Krishna stated and sketched a proof over infinite fields. But he didn't have time to fully prove it. He did prove Nullstellensatz using Noether normalization, to give an idea of its power. We did many of the exercises in the fifth chapter of Atiyah-Macdonald during the tutorials.

The tutors Mr. Suhas B, Ms. Mitra Koley and Mr. Praveenkumar Roy were very much helpful during the tutorials.



Schedule of Lectures and Tutorials

Day	Date	Lecture 1 9:30- 11:00	11:00- 11:30	Lecture 2 11:30- 13:00	13:00- 14:30	Tutorial 1 14:30-15:30	15:30 - 16:00	Tutorial 2 16:00-17:00	17:00- 17:30
Mon	24/4/17	MK	Tea	SS	Lunch	MK+Tut1+Tut2	Tea	SS+Tut1+Tut2	Snacks
Tues	25/4/17	MK		SS		MK+Tut1+Tut2		SS+Tut1+Tut2	
Wed	26/4/17	MK		SS		MK+Tut1+Tut2		SS+Tut1+Tut2	
Thu	27/4/17	MK		SS		MK+Tut1+Tut2		SS+Tut1+Tut2	
Fri	28/4/17	MK		SS		MK+Tut1+Tut2		SS+Tut1+Tut2	
Sat	29/4/17	MK		SS		MK+Tut1+Tut2		SS+Tut1+Tut2	
SUNDAY : OFF									
Mon	1/5/17	CD	Tea	KH	Lunch	CD+Tut3+KH	Tea	KH+Tut3+CD	Snacks
Tue	2/5/17	CD		KH		CD+Tut3+KH		KH+Tut3+CD	
Wed	3/5/17	CD		KH		CD+Tut3+KH		KH+Tut3+CD	
Thu	4/5/17	CD		KH		CD+Tut3+KH		KH+Tut3+CD	
Fri	5/5/17	CD		KH		CD+Tut3+KH		KH+Tut3+CD	
Sat	6/5/17	CD		KH		CD+Tut3+KH		KH+Tut3+CD	

List of Participants

Sr.n	Full Name	Affiliation	Position in College/ University
1	Mr. Ravi Shankar Kapildev Yadav	SSR College of Artrs Commerce and Science	Asst. Prof.
2	Mr. N. Annamalai	Bharathidasan University	PhD
3	Mrs Viji M.	St. Thomas' College, Thrissur	Asst. Prof.
4	Ms Meera Sn	Manonmaniam Sundaranar University	PhD Student
5	Mr. Amit Sharma	Pratap Institute Of Technology And Science	Asst. Prof.
6	Prof. Dipak Sandu Jadhav	Smt. Chandibai Himathmal Mansukhani College	Asst. Prof.
7	Mr. Harish V N	Sree Kerala Varma College	Asst. Prof.
8	Mr. V. Ramanathan	Manonmaniam sundaranar University, Tirunelveli	Research Scholar
9	Mr. Gaurav Shantaram Jadhav	V.P. Varde College of Commerce & Economics	Asst. Prof.
10	Mrs Reshma E	Payyanur College Payyanur	Asst. Prof.
11	Mr. Subhash Mallinath Gaded	R K Talreja College of Arts, Science & Commerce, Ulhasnagar-03.	Asst. Prof.
12	Mr Raveesh R Varrier	ST.ALOYSIUS COLLEGE	Asst. Prof.
13	Ms Akshara R	St ALoysius College	Guest Lecturer
14	Prof. Pooja Gurmukhdas Rajani	Smt. Chandibai Himathmal Mansukhani College.	Asst. Prof.
15	Ms Athira Satheesh K	Payyanur college	Guest lecturer
16	Ms. Chathely Briji Jacob	Birla College of Arts, Science and Commerce.	Visiting Faculty
17	Mr. Sushobhan Maity	Visva-Bharati	PhD
18	Mr. Kamalakar Ramesh Surwade	University of Mumbai	Asst. Prof.
19	Mrs. Deepthi A N	Sree Narayana College	Asst. Prof.
20	Mrs. Sabna K S	KKTM College, Kodungallur	Asst. Prof.
21	Mrs. Sherin Jose T	St.Joseph's College, Irinjalakuda, Thrissur, Kerala	Asst. Prof.
22	Mrs. Sinda Joy	St.Joseph's College, Irinjalakuda, Thrissur, Kerala	Asst. Prof.
23	Ms. Dhanya V S	St.Joseph's College, Irinjalakuda, Thrissur, Kerala	Asst. Prof.
24	Dr. Sr. Deeni C J	St.Joseph's College, Irinjalakuda, Thrissur, Kerala	Guest Faculty
25	Sonadas P	St.Joseph's College, Irinjalakuda, Thrissur, Kerala	Guest Faculty
26	Shajila K Y	St.Joseph's College, Irinjalakuda, Thrissur, Kerala	Guest Faculty

Sherin Jose T

Instructional School for Teachers in "Commutative Algebra"

24th April – 06th May, 2017

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Valedictory



The valedictory function was presided over by Dr. Sr. Lilly P L, Principal, St. Joseph's College [Autonomous], Irinjalakuda. The certificates were distributed during the function. The speakers and participants shared their experience. This programme was ended with a high-tea.